Ethiopic Easter Computus

by

Otto Neugebauer

To the Memory of Eduard Schwartz (1858-1940)

1. "Easter" being defined as the first Sunday after the first Full Moon after the vernal equinox would require, if taken astronomically seriously, not only the determination of the length of the tropical year but also a solution of the highly complex problem of predicting the moments of the full moons.

Historically this intricate definition originated from the connection of the Christian feast with the Jewish Passover date, the 14th of Nisan, i.e. with a lunar calendar, that is to say a "year" based on lunar months whose first days are the days of first visibility of the new crescent. As is well known these days were determined in Jerusalem, before the destruction of the Temple, by direct observation. The "full moon" was then schematically defined as the 14th day of the lunar month (in good Babylonian tradition) and the relation of this lunar calendar to the solar year was regulated (again following Babylonian example) by using a 19-year cycle in which twelve "years" were given 12 lunar months, while the remaining seven ("intercalary") years had 13 lunar months. The resulting pattern indeed keeps the beginning of a given month in a fixed neighborhood of the vernal equinox, i.e. it prevents the lunar year from "rotating" with respect to the "solar year". Hence the determination of Passover in Jerusalem had been a simple affair. For the Jews in the Diaspora, however, the situation was quite different. Direct observation of the new crescent from some other locality, e.g. Alexandria or Rome, need not result in identical dates with Jerusalem, nor could one ignore the existence of a civil calendar which regulated the lives of the majority of the population.

One way out of this dilemma would consist in applying the best available astronomical theory of the solar and lunar motion and of the lunar visibility for given geographical conditions to the determination of the evenings of first lunar visibility. One such highly sophisticated attempt is well known to us. In the late 12th century Maimonides discussed this problem on the basis of the Ptolemaic lunar theory (for the accurate determination of the conjunctions of sun and moon) combined with a theory of lunar visibility closely reminiscent of Babylonian methods known from the Seleucid-Parthian period (though not identical in details).

For the early Christian period we have no treatise that would inform us about the theoretical background of the Easter computus. We know much about the "Easter Controversy" between Roman and Alexandrian dates and we know, from the time of Constantine on, of the frequently expressed condition for the Christian feast to avoid any coincidence with the Jewish Passover. But we know pratically nothing about the festival calendar of the Alexandrian Jews during the early centuries of Christianity.

It is at this point that it seems reasonable to look at Ethiopic sources. We know of the existence of large tables displaying dates for Easter and Passover and of numerous related treatises, from a few paragraphs in length to many folios of diverse contents. Since the Ethiopic calendar is identical with the Alexandrian (as established by Augustus) one may well hope to find here also Hellenistic-Roman material preserved in the Ethiopic isolation, as is the case with various religious literature (e.g., the "Book of Enoch" and similar works). Many modern scholars have expressed their opinion that the Alexandrian Easter computus represents the last flowering of "Alexandrian Science", which indeed reached during the early Christian centuries its highest development as far as astronomical science and methodology is concerned. The second century saw the publication of Ptolemy's "Almagest", and around 400 the "Handy Tables", in Theon of Alexandria's version, appeared — two works which constituted the basis of all Arabic and Western astronomy of the Middle Ages. Indeed, it would be worthwhile to establish the connection between the origin of the Christian Easter Computus and the contemporary Alexandrian astronomical techniques, well known to us from excellently preserved sources.

Some fifteen years of increasingly detailed study of Ethiopic sources have led me to a very different evaluation of the situation. Not that it could be doubted that the Ethiopic tables and treatises reflect the Alexandrian computus of early Alexandrian Christianity. But there is no trace of Alexandrian "science" in this whole procedure, which turns out to be of the utmost simplicity. Its foundation, however, is the (equally simple) Jewish Passover computus which is nothing but a simple adaptation to the Alexandrian calendar of the 19-year cycle of a schematic lunar calendar. One single rule determines the Alexandrian Easter Computus: Easter is the first Sunday after Passover. Since Passover is by definition a "full-moon" date, Easter follows the full moon, and a proper definition of the date of the "Vernal Equinox" in the Jewish 19-year cycle introduces automatically the corresponding Christian rule. In return for discarding the myth of Alexandrian science in the Easter reckoning, we obtain a clear picture of the festival calendar of the Alexandrian Jews during the early Christian period. This calendar is also of the utmost simplicity, exclusively based on a 19-year cycle of the most elementary structure (and not to be taken as the equivalent of its Babylonian ancestor). The Ethiopian texts have indeed provided new insight into Alexandrian calendaric conditions but only at the price of disspelling the traditional picture of a connection between early Christianity and contemporary pagan science.

2. In order to see our discussion of the Easter computus in the proper perspective, it will be useful to sketch the background of Ethiopic "astronomy" in general. We know of only two groups of problems which have no direct relation to the calendrical tables: one concerns the reckoning of hours of daytime by means of the shadow cast by a man standing upright; the other tabulates the rising amplitudes of the moon during 12 consecutive months of a lunar year. Both of these problems are dealt with by the simplest arithmetical patterns. For example, the noon shadow in 7 consecutive months is supposed to be (measured in feet) 2 3 4 5 6 7 8 and then back again from 7 to 3 in the remaining 5 months. In each month the hours before and after noon are found from the noon shadow by adding 1, 2, 3, 4, and 10 feet to its predecessor (thus, e.g., for the first six hours 22, 12, 8, 5, 3, 2 before noon in the first case). Since in all cases the day is divided into twelve hours, these hours are "seasonal hours", a common type in Alexandrian Egypt. On the other hand the same treatises that contain shadow tables operate also with "equinoctial hours" by assigning variable lengths of daylight to each month, either in the ratio 15:9 or 2:1. These internal contradictions are of only minor significance, however, as compared to the fact that the shadow lengths of such ratios are totally excluded for Ethiopia with its almost equatorial latitude. It must be admitted, however, that similar tables were copied, century after century, from Byzantium to monasteries in northern France, without the slightest chance for practical usefulness and at a time when the correct determination of the variation of the length of daylight and of shadow lengths had long been found in Greek astronomy.

Also the "gates" of moonrise are determined by similar arithmetical patterns. From the "Book of Enoch" and from the Dead Sea Scrolls, it seems likely that these schemes originated in the Palestinian area some time in the last centuries before our era. In the present context it is of interest that the "months" during which the moon traverses once back and forth six sections of the horizon (the "gates") add up, alternatingly, to 30 and 29 days. Here we have a typical example of a schematic "lunar year" of 6 "full" and 6 "hollow" months, thus 354 days in length. Again, this is a widespread schematic description of the variability of the synodic months, convenient for use but far removed from the actual facts which were analyzed in great detail and with remarkable success in the cuneiform ephemerides computed in Mesopotamia during the centuries from about — 300 into the first century A.D. And again, the basically correct analysis of the variability of the lunar months and their application to the Babylonian lunar calendar had not the slightest effect either on the Greek lunar calendars or on the "Gates" and other concepts in the Book of Enoch and similar compositions of enormous popularity.

3. A short description of the Ethiopic sources of our study seems desirable in the present context. I know of only one consistent work on calendaric-chronological matters : the computus by Abū Shākėr, a book of 59 chapters, written in the 13th century in Egypt and translated in the 16th century from the Arabic original (now lost) into Ethiopic. It has never been edited or translated — a fate shared with almost all of the other calendaric sources — in spite of the fact that it is considered to be the basis of all Ethiopic "computus". References to it are not rare in other short treatises but without much justification, as far as I can see. Another famous computus was supposedly written in A.D. 213/14 by Demetrius, the 12th Patriarch of Alexandria. Actually there is no treatise preserved which would safely be related to one author. Finally, some late additions to our material are taken (usually out of context) from Arabic astronomy, always written in Amharic, and without any influence on the traditional treatises.

Excepting Abū Shākėr, the texts which concern the computus consist of a chaotic mixture of short sections that deal more or less directly with concepts needed for the construction of the tables whose final goal is the determination of the dates of all moveable feasts, Jewish as well as Christian. One could imagine that we have here countless scattered fragments and excerpts from some larger treatise explaining the structure and usage of the calendarical tables, somewhat similar in purpose to the introductions to the "Handy Tables" in Greek astronomy.

This material, as we have it, consists of many variants of texts, many times senselessly distorted by repeated copying, and usually not understood by the scribes. The general tendency is "didactic", i.e. the mechanical compilation of rules which ordinarily are simple consequences of another rule formulated in some other paragraph a little earlier or later. The chaos is increased by the desire to incorporate into sections based on the Alexandrian calendar and the Jewish Passover computus also the wisdom of the "Enoch" tradition, that means to consider "years" of 364 days, or "seasons" of 91 days each. Later scribes might then improve on such passages by adding a new layer of Julian data onto Enochian passages — the so-called "Slavonic Enoch" shows nice examples of this process which has also bewildered modern commentators.

In spite of this chaos of fragmentary treatises, it is quite possible to bring sense into the methods which were used for the computation of the tables and for the control of the numerical data. Having once understood the structure of the tables, the bulk of the texts makes sense, even if marred by a lack of distinction between a few basic rules and a host of rather obvious consequences which any user could have derived by himself*.

4. Before turning to a description of the Ethiopic tables it is convenient to mention a mathematical terminology (introduced in 1801 by Gauss, who, by the way, also wrote an article about the numbertheoretical structure of the modern Easter canon, based on the "reform" of 1582 under Gregory XIII).

We say that two numbers a and b are "congruent modulo c" (written $a \equiv b \bmod c$) if the difference $a - b$ is divisible by c. For example, $39 \equiv 1 \bmod 19$ because $39 - 1 = 2 \cdot 19$. In particular $a \equiv 0 \bmod c$ means that a is a multiple of c; e.g., $76 \equiv 0 \bmod 19$ but also $76 \equiv 0 \bmod 4$ (because $76 = 19 \cdot 4$).

Almost all calendaric operations can be conveniently expressed as "congruences". For example, the "Enoch-year" is 364 days long and $364 \equiv 0 \bmod 7$. This implies that the position of the weekdays remains always the same in this type of year. Obviously this was the very purpose of creating such a year and all our texts confirm that it was never modified. Modern scholars tried to discover some hidden intercalation system because they could not imagine that one could live with a "rotating" calendar. Apparently they do not know, e.g., about the rapidly rotating lunar calendars of the Assyrians or of the Islamic calendar.

The "Egyptian year" of 365 days is congruent 1 mod 364. Consequently the Ethiopic texts speak of an "extra" day when one goes from an Enoch year to a 365 day year. But $365 \equiv 5 \bmod 30$, hence the 5 days in excess of 12 civil months of 30 days each are called epagomenal days. Finally, $365 \equiv 1 \bmod 7$. Consequently if, e.g., Jan. 10, 1978 was a Tuesday then this same date in 1979 will fall on a Wednesday, in 1980 on a Thursday. But the year 1980 has 366 days, $\equiv 2 \bmod 7$; thus the next year Jan. 10 will jump to Saturday.

Four Alexandrian (or four "julian") years total a number of days which are congruent $3 \cdot 1 + 2 = 5 \bmod 7$ and since $5 \equiv -2 \bmod 7$ we can also say that weekdays in the Alexandrian calendar recede 2 days in each quadruple of years.

These different forms of "years" are intended to agree more or less with the "solar year" i.e. with the climatic seasons. "Lunar years", however,

* For details see my monograph "Ethiopic Astronomy and Computus" (Oesterr. Akad. d. Wiss., Phil.-Hist. Kl., S.B. 347, 1979).

can produce such an association only by switching occasionally from a year of 12 months to a year with 13 months. Consequently such years, operating either with accurate lunar months (as the computed Babylonian months) or with schematic months (of 29 or 30 days) cannot produce "years" of fixed lengths.

The months themselves in the years of Enoch, in the Egyptian and in the Alexandrian calender, are no longer related to the moon but are fixed at 30 days of length. Only the theoretically determined lunar months of the Babylonian ephemerides vary between full and hollow according to the highly complicated factual variation of the dates of first visibility. Indeed these data are by no means simple (or even regular) alternations between full and hollow months. And since the character of each month depends on the moon's visibility that defines the first day of a month the Babylonian calendar days begin in the evening. This "evening epoch" of "lunar days" was also taken over by the Jewish calendars. The Egyptian days, however, and with it the days of the Alexandrian calendar, are counted in "morning epoch". This then has also become the norm in the Ethiopic tables.

We have colophons in Ethiopic texts, or dates in documents or annals, which give two days. A book may have been finished, according to a literal translation of the colophon, "on the 6th at the beginning of night, at the 10th at the beginning of day". Such obvious nonsense has disturbed few translators. In fact, "beginning of night" must mean "days which begin in the evening", i.e. simply "lunar dates" in contrast to the Alexandrian dates in morning epoch or simply "civil dates". We have extensive rules in our texts on how to find the lunar dates from civil dates, and vice versa. All Jewish feasts have not only civil dates but also lunar dates, in particular Passover has the lunar date 14, i.e. (schematic) full moon. If p is the civil date of Passover, f of Easter, then the lunar date of Easter is simply $14 + f - p$. In mediaeval terminology this is the "luna", the "age of the moon", of Easter which by definition must be more than 14 since f must be later than p. It is one of the points of controversy in the contest between Alexandria and Rome whether it is permissible that the luna of Easter is as low as 15 (Alexandrian norm), i.e., that Easter can be a Sunday following a Passover that falls on a Saturday. The Ethiopic tables show that they followed the Alexandrian norm in giving dates as low as $f = p + 1$.

5. Babylonian astronomy is built on the experience that astronomical phenomena repeat themselves periodically. Lunar eclipses, for example, return in the same magnitude in a cycle of 18 years. Saturn returns to the same region among the fixed stars in about 30 years, Jupiter in 12. Consequently these two planets will be in the same position relative to each other in $2 \cdot 30 = 5 \cdot 12 = 60$ years. By combining characteristic periods in this way it is easy to predict (or to exclude) situations of a more complex character. For a people who lived with a real lunar calendar it was only natural to observe also the return of the new moon, that is, the conjunction of sun and moon, to the same position in the sky. The result then is a number of "synodic" months that corresponds to a number of ("sidereal") years. It turns out that with a high degree of accuracy 235 is this number of months. Since $235 = 19 \cdot 12 + 7$ one can add 7 "intercalary" months to 19 ordinary "lunar years" and will obtain agreement with 19 "solar" years. This interval is called the "19-year cycle" or the "Metonic cycle" (because it was proposed, perhaps independently — and unsuccessfully — by Meton in Athens). We shall meet a simplified version of this cycle in the Ethiopic tables.

A cycle without any real astronomical background is the 7-day week. If we combine it with the 4-year cycle of the Alexandrian intercalation, we see that only after $7 \cdot 4 = 28$ years an Alexandrian year will begin with the same weekday. In our treatises this cycle is called the "solar cycle". If we wish to combine weekdays, Alexandrian calendar, and lunar phases, we must seek a common period of the solar cycle with the 19-year cycle. Since 19 and 28 have no common factor, the shortest period which comprises 7, 4, and 19 is the product 532 of these three periods. This number 532 is at the foundation of the whole Easter computus.

The 532-year cycle is well known to the mediaeval computists, from the Greek East to the Latin West. When the Monk Dionysius Exiguus in the middle of the 6th century introduced our present era he related the year 532 of his new "Christian era" with the then current era of Diocletian by equating A.D. 532 with Diocletian 248. The Ethiopic eras, based on Alexandrian prototypes are arranged slightly differently. The era W of the "World" (or "from Adam") is related to the era J of the "Incarnation" by the relation J 0 = W 5500. The era of the Incarnation is connected with the Era Diocletian (or the "Era of the Martyrs") by D 0 = J 276 (hence D 248 = J 524 and J 0 corresponds to A.D. 7/8). Finally, an era of "Grace" or "Mercy" is defined by G 0 = D 76. The reason for this norm of the most commonly used era is simply that G 0 = W 5852 $\equiv$ 0 mod 532. Hence the beginning of the era G coincides with the beginning of a 532 year cycle (the 12th) of the era W. But all these eras are based on the era Diocletian and thus on the Alexandrian calendar as established by Augustus.

6. We now can turn to the Ethiopic calendaric tables. Their most important type, preserved in many copies (but unpublished) consists of 28 tables, each of which covers one 19-year cycle; we therefore call these tables the "532-year tables". They are usually based on the era G and therefore concern one of the three cycles that begin ($k = 1$) with the years W 5853, 6385, and 6917.

All of the existing manuscripts were written in the last, 14th, cycle (from A.D. 1424/5 into the 20th century). There exist several types of shorter tables, all of which have periods of 19 years, or of multiples of 19, and are therefore implicitly contained in the 532-year tables, though in different arrangement, e.g. by weekdays.

The main type of the 532-year tables contains about 20 columns but there exist larger tables with about 30 columns. Most of these tables contain a first column, headed "tārik", i.e. "history". The next two columns count the lines either from $c = 1$ to 19 in each individual table or from $k = 1$ to 532 in the whole set. The column "tārik" mentions events of Biblical history or of contemporary history, (e.g. the death of Patriarchs or Kings), without formal distinction of these dates with respect to the cycle to which they belong. For example, the entry "baptism" at $k = 211$ refers to the baptism of Christ in the year W 5531 = G 211 (in the 11th cycle).

c	e	m	yk	tb	p
1	0	30	9	14	10
2	11	19	28	3	*29*
3	22	8	17	22	18
4	3	27	6	11	7
5	14	16	25	30	*26*
6	25	5	14	19	15
7	6	24	3	8	4
8	17	13	22	27	23
9	28	2	11	16	12
10	9	21	30	5	1
11	20	10	19	24	20
12	1	29	8	13	9
13	12	18	27	2	*28*
14	23	7	16	21	17
15	4	26	5	10	6
16	15	15	24	29	*25*
17	26	4	13	18	14
18	7	23	2	7	3
19	18	12	21	26	22

"Table XIX"

The remaining columns refer to the dates of feast days, Jewish and Christian, culminating in the last column "fāsikā", i.e. "Easter". Several columns show for all years the same number and are headed "beginning of night", i.e. lunar date. They always belong to a neighboring column which gives the civil dates of a Jewish feast. Thus the lunar date of maṭqeʿe ("trumpet", i.e. the Jewish New Years Day) is 1, of Yom Kippur 10, of Tabernacle 15, and of

Passover 14. Looking more closely at the civil dates of the Jewish feasts one will notice that they are the same in each 19-year table. Excerpting these data from the larger tables we obtain the above shown "Table XIX" which is repeated 28 times. Here e denotes the "epact" which is related to the date m of the New Year by $e + m = 30$. All numbers in the last four columns increase by 19 (mod 30) every year, hence e must decrease by 19, or modulo 30, increase by 11. A date of Passover printed in italics indicates that it belongs to month VII of the Alexandrian calendar (Phamenoth = Ethiopic Magābit). All other Passover dates belong to VIII (Pharmouthi = Miyāzyā). It follows from the arithmetical structure of Table XIX that the dates of all Jewish feasts are known as soon as the date of one of them is known. For example, $m \equiv p + 20$, $yk \equiv p - 1$, $tb \equiv p + 4$, always mod 30.

It should furthermore be noted that the periodicity of this table requires that the transition from the line $c = 19$ to $c = 1$ requires for e the addition of 12 days instead of the usual 11. Correspondingly all other dates increase by only 18 instead of the ordinary 19. This specific situation is described by the medieval computists as the "saltus lunae", the object of much empty speculation. In fact it represents a very simple matter. A schematic lunar year has a length of 354 days, hence receding 11 days each year with respect to 365 days. These 11 days are called the "epact" in Greek and Western medieval astronomy. Continued application would remove a lunar date, e.g. m of the Jewish New Year, from its general location in the solar year and thus a full month of 30 days will be added. This explains our sequence m in Table XIX in which we add the proper month numbers of the civil calendar:

$e = 0$	$m =$ I 30	$p =$ VIII 10	$c = 1$
11	I 19	VII *29*	2
22	II 8	VIII 18	3
3	I 27	VIII 7	4
14	I 16	VII *26*	5
25	II 5	VIII 15	6

This also shows that the rule $p \equiv m + 10$ mod 30 results from a fixed distance $p = m + 190$. The same holds for all moveable festivals. It is the same to say that the dates of m are restricted to the interval I 15 $\leqq m \leqq$ II 13 and, similarly, Passover to VII 25 $\leqq p \leqq$ VIII 23.

Incidentally it may be remarked here and for all that follows that it is of primary importance to express all arithmetical rules in the system in which they were developed, i.e., in the Alexandrian calendar with its 30-day months. Introducing our "julian" calendar with its perverse disorder of month-lengths completely obscures the arithmetical simplicity of all structures.

7. The above rules for the determination of the civil dates of the Jewish festivals give us a complete insight into the meaning of "19-year cycle" in this procedure. In Babylonian astronomy the 19-year cycle assumed that 19 sidereal years are equal to 235 mean synodic months. In our present tables, however, 19 Alexandrian years are equated with 235 schematic lunar months. Furthermore, in the Babylonian calendar the months followed closely the complicated pattern of the intervals between evenings of first visibility. The Ethiopic cycle knew nothing of such refinements. It simply assumed an epact of $11 = 365 - 354$ days which was a crude but convenient estimate for the slippage of the lunar phases, and adjusted this 11-day epact (with the help of the "saltus lunae") so that it returned to the same civil dates after 19 Alexandrian years without concern for the location of the Alexandrian intercalations. Since this scheme is extended over 532 years exact periodicity is granted also with respect to intercalation. In fact, this is already the case after $4 \cdot 19 = 76$ years, an interval which appears frequently in calendaric treatises. If the interest is centered on weekdays $7 \cdot 19 = 133$ years are significant. But 532 years remains as the shortest cycle for all parameters under consideration. If one wishes to convince oneself of the quality of this cycle, one can remark that $532 \cdot 365;15$ days are assumed to be equal to $28 \cdot 235$ synodic months, which gives for one month the length of about 29;31,51,4 days, which is a very good approximation. This illustrates the fact that very good results can be reached (often accidentally) by extremely simple arithmetical procedures.

Modern scholars cherished the idea that occasional "observations" of full or new moons were applied to "correct" the results of cyclic computation. The high quality of the approximation of the cycle during five centuries shows that such empirical corrections were not at all necessary. On the contrary: the occasional comparison with some true conjunction or opposition would have only introduced errors to the full amount of the considerable difference that can occur between "mean" and "true" syzygies.

8. Having reached complete insight into the pattern for the dates of the Jewish festivals, we can obtain the same for the Christian feasts without further difficulties. Exactly as in the preceding case all Christian dates are known from any one of them. For example, "Beginning of Fast" $\equiv$ "Nineveh" (n) + 14; "Mount Olive" $\equiv n + 11$; "Palm Sunday" $\equiv n + 2$, and Easter (f) $\equiv n + 9$, always modulo 30. The proper months are determined from respective limits, for example VII 26 $\leqq f \leqq$ VIII 30.

The really crucial rule concerns Easter. It is simple enough: Easter is the Sunday following Passover. Since p was limited by VII 25 as the earliest date and since VII 25 is considered to be the date of the vernal equinox (how accurate astronomically is of no concern), we have now established Easter in the canonical fashion : after equinox, after full moon (i.e. Passover) and a Sunday.

We thus see that the whole Christian calendar was made dependent on the Passover date, which in turn is a simple application of the epact computus of the schematic "19-year cycle". It is indeed as a text expressed it : "maṭqeʿe and epact are the foundation of the whole computus".

There remains only one little step to be clarified : obviously we need now to know the weekday of Passover. But since Passover = maṭqeʿe + 190 and since $190 \equiv 1 \bmod 7$, it suffices to determine the weekday of m. But the weekday of any Alexandrian year, or of a year of the era Diocletian, has been known since Antiquity. Hence our table simply lists, in a column headed "ṭentyon", the weekdays of the first of Thoth. For example, we have for the first day of each 532 year cycle of the era G the weekday "Tuesday". Since column m gives us the civil date of the maṭqeʿe we can immediately determine the weekday of m and thus of Passover and finally the date of the next Sunday. This solves our problem.

9. Example : find the date of Easter for the year $k = 118$ in the era G. Since $118 \equiv 4 \bmod 19$, we have $c = 4$; hence (from Table XIX) $m =$ (I) 27 and $p =$ (VIII) 7. The 532-year tables give for $k = 118$ Monday as the weekday of I 1. Now I 27 = I 1 + 26 and $26 \equiv 5 \bmod 7$; thus weekday of m = Monday + 5 = Saturday. Hence the weekday of p = Saturday + 1 = Sunday. And hence Easter is 7 days later, i.e., $f = p + 7 =$ VIII 7 + 7 = VIII 14; the "luna" of Easter Sunday is $14 + 7 = 21$.

Check with modern tables : $k = 118$ corresponds to W 6916 + 118 = W 7034 = J 1534 = A.D. 1542. For this year Alexandrian VIII 14 = April 9, which is indeed the Easter Sunday for 1542. The preceding astronomical full moon, however, was on March 31, i.e. 2 days before the date of Passover.

In the above computation we used only the basic elements of the 532-year tables. Many of these tables give, however, columns both for the weekday of m and of p. Hence we would have seen from the table that $p =$ VIII 7 fell on a Sunday, and hence $f =$ VIII 14. Only after the Gregorian reform in 1582 would the Ethiopic tables no longer be useable for the determination of the Catholic Easter dates.

10. In principle we have now reached our goal to explain the method by which the Ethiopic 532-year tables furnished the dates of Easter year after year. It is the purpose of the subsequent sections to discuss the historical background of these tables and related treatises.

First of all we should elaborate somewhat on our main result — the location of Easter Sunday in the week immediately following Passover. The correctness of this relationship can be demonstrated in three ways: first, by passages in our calendaric treatises stating explicitly this rule; secondly, by purely arithmetical proof on the basis of the structure of the relevant columns; thirdly, by simply exerpting from the tables the date f of Easter as a function of the date p of Passover. The result of this last, most direct proof is shown in the subsequent table. Column c gives the cycle number in each of the 28 19-year cycles; column p is the same as in our previous "Table XIX" (p. 94); column f shows all attested Easter dates correlated with the same pair of number c and p. Obviously f ranges from $p + 1$ to $p + 7$, i.e., the space of one week after $p + 1$ as required by the fundamental rule, which is thus fully demonstrated.

c	p	f						
1	10	11	12	13	14	15	16	17
2	*29*	*30*	1	2	3	4	5	6
3	18	19	20	21	22	23	24	25
4	7	8	9	10	11	12	13	14
5	*26*	*27*	*28*	*29*	*30*	1	2	3
6	15	16	17	18	19	20	21	22
7	4	5	6	7	8	9	10	11
8	23	24	25	26	27	28	29	30
9	12	13	14	15	16	17	18	19
10	1	2	3	4	5	6	7	8
11	20	21	22	23	24	25	26	27
12	9	10	11	12	13	14	15	16
13	*28*	*29*	*30*	1	2	3	4	5
14	17	18	19	20	21	22	23	24
15	6	7	8	9	10	11	12	13
16	*25*	*26*	*27*	*28*	*29*	*30*	1	2
17	14	15	16	17	18	19	20	21
18	3	4	5	6	7	8	9	10
19	22	23	24	25	26	27	28	29

It should be noted that our table gives only $7 \cdot 19 = 133$ values for f. Hence each Easter date must occur four times in 532 years. The explanation of this multiplicity lies, of course, in the fact that our rule does not contain any statement about the Alexandrian intercalation which produces four different possibilities for each combination p, f.

If one investigates the occurrence of these four cases within the 532-year tables one finds (either by arithmetical theory or by inspection) an important phenomenon: these cases are always $95 = 5 \cdot 19$ years apart. For example, the combination $p = 10, f = 11$ occurs in the years $k = 115, 210, 305$, and 400. Similarly, the earliest possible Easter date $f =$ (VII) 26 (at $c = 16, p =$ (VII) 25) occurs in the years $k = 54, 149, 244$, and 491 which is $\equiv 54 - 95$ mod 532.

The author (or authors) of the 532-year tables was fully aware of this law of distribution for equivalent values within a 532-year cycle and statements to this effect are also found in our treatises and will not surprise anyone who actually computes a complete 532-year table. Medieval Latin computists were also aware of this "periodicity" which explains why, for example, the tables of Dionysius cover the 5 19-year cycles from A.D. 532 to 626. Modern writers on medieval computus missed this point, stating correctly that 95 is not a period in the 532-year cycle but ignoring the fact of the unavoidable multiplicity of data in groups of 95 years.

Recognition of the 95-year intervals is not the only procedural element that spilled over from the Alexandrian computus to the Latin one. As we have seen, the determination of Easter Sunday requires knowledge of the weekday of the first of Thoth (śarqatito in Ethiopic). As noted before, this day is given in our tables in the column headed "tentyon" (t) (which is a distortion of the term [ἡμέραι] τῶν θεῶν, used for "weekday", e.g., by Athanasius). This number counts the weekdays (modulo 7) so that 1 = Wednesday. Latin scribes, however, used a norm for the "feria" in which 1 = Sunday. Now it so happens that $t = 1$ (Thoth = Maskaram 1 Wednesday) always corresponds to March 24 = Sunday = feria 1. Thus all rules that involve t are numerically identical with rules which use the feria of March 24, a number which the medieval computists honored with the special name concurrentes.

11. Our calendaric treatises are full of invective against the "impious Jews", stressing over and over again the purpose of the rules concerning Easter to avoid contamination by Passover. Nevertheless they allowed, as we have seen, an approach to the very next day. The "Romans", eager to follow rules of their own, and opposed to Alexandrian superiority, insisted on a two-day minimum, i.e., on a lowest "luna" 16 as against the Alexandrian 15. Of course neither one of these norms has any astronomical basis whatsoever and is simply a matter of arbitrary choice for the boundaries of a parameter.

This is not the only object in the bitter Easter controversy between Alexandria and Rome. Since the Alexandrians had the good luck to adopt the Jewish 19-year cycle, the Romans insisted on some other cycles based mainly on the "octaeteris" which relates 8 years to $8 \cdot 12 + 3 = 99$ months, corresponding to a mean synodic month of about $29;30,54^{d}$, which by any standard of ancient astronomy is of clearly inferior accuracy. Hence the necessity of repeated corrections of the cycle—not, of course, by "observation" but by adjustment to the Alexandrian norm. It was only in the sixth century that Dionysius broke the impasse by accepting the Alexandrian pattern on the authority of a (spurious) decree of the Council of Nicaea, and by replacing the simple data of the Alexandrian calendar by the Roman calendar with all its pagan relics of calends, ides, and nones. Furthermore, by transforming the years of the Diocletian era to the years "A.D." he became the father of our present calendaric system.

12. There remains one more point to clarify. We have repeatedly referred to Alexandrian procedures from evidence in the Ethiopic tables and treatises. To what extent are we justified in doing so, even if it would appear *a priori* unlikely to assume Ethiopic innovations in these texts which abound in Greek terminology and concepts? One must nevertheless admit that our data are chronologically fixed for only mod 532 years and it could be possible that any one of these cycles represents historically the first one.

Here a lucky accident comes to our rescue. In 1976 Ephraim Isaac (of Harvard University) published a catalogue of Ethiopic manuscripts in the library of the Armenian Patriarchate in Jerusalem. This catalogue mentioned among others two manuscripts of evident interest to our discussion. One was obviously a shadow table of a well-known type; the other manuscript suggested a 532-year table. By courtesy of His Eminence, the Patriarch, I received photographs of these manuscripts which confirmed my initial conjecture concerning their contents. But the 532-year table contained an unexpected variant beyond some slight changes in arrangement: it contained a column giving the "indictio" of the year.

As is well known, this parameter refers to a 15-year cycle introduced by Diocletian for administrative purposes. But somehow this number acquired the role of a short-term era, frequently used in all kinds of documents from Byzantine domains to the medieval West. The application of this count also in the 532-year tables is of primary interest to our problem. Since 15 has no common divisor with 4, 7, or 19, it repeats itself only with the same line of data in $15 \cdot 532 = 7980$ years. In other words: the indictio listed in one of our tables fixes its date uniquely.

The time scale of the indictions is of course well known from ancient and medieval documents. For us it is enough to mention that Athanasius regularly gives the indictio of the year in his Easter messages. The subsequent table gives in its upper section a transcription of the Ethiopic table.

	D	i	ep	t	j	e	m	$\overline{m}$	yk	$\overline{yk}$	tb	$\overline{tb}$	bf	$\overline{bf}$	p	$\overline{p}$	pw	f	$\overline{f}$
Jerus. Arm. 3483 194v/195r	44	1	6	1	4	25	5	1	14	10	19	15	24	22	15	14	4	19	18
	45	2	5	2	5	6	24*	1	3	10	8	15	16	25	4	14	1	11	21
	46	3	5	3	6	17	13	1	22	10	27	15	29	19	23	14	7	24	15
	47	4	5	4	7	28	2	1	11	10	16	15	21	22	12	14	4	16	18
	48	5	6	6	2	9	21*	1	30*	10	5	15	12	24	1	14	2	7	20
	49	6	5	7	3	20	10	1	19	10	24	15	2*	25	20	14	1	27	21
	50	7	5	1	4	1	29*	1	8	10	13	15	17	21	9	14	5	12	17
Athanasius	44	1		1		25												19	18
	45	2		2		6												11	21
	46	3		3		17												24	15
	47	4		4		28												16	18
	48	5		6		9												7	20
	49	6		7		20												20	15
	50	7		1		1												12	17

We need not describe the single columns since all we need are the parameters already defined in the preceding pages. Only the last column $\bar{f}$ should be mentioned since it contains the values of the "age" of the moon, i.e. the "luna" $14 + f - p$. The era in the first column is the era D of Diocletian. Changing it to the era G = D − 76 one obtains exact agreement of all columns with the ordinary 532-year tables. But when this agreement normally would give only the date of these tables modulo 532, we can now say — because of the presence of a column with the indictio (i) — that D 44 can only mean the year Diocletian 44. But exactly for these years we have also the elements quoted by Athanasius: the years of Diocletian, the indictio, the "ṭentyon", i.e. the weekday of I 1, the epact e (thus also $m = 30 - e$), the dates of Easter (f) and the age of the moon ($\bar{f}$), thus also the date of Passover $p = 14 + f - \bar{f}$.

Hence Athanasius' dates give all elements underlying both the Jewish and the Christian calendars that are necessary for the determination of Easter. And it is now rigorously proved that the Ethiopic tables are identical in substance with the Alexandrian Easter computus of the time of Athanasius.

13. One may rightly say that this result is not surprising, though one may also remark that there is always a certain difference between historical plausibility and a mathematical proof that does not imply anything but numerical data, comparable to the data of a sharply defined solar eclipse.

But we also have gained independent historical information. Knowing now in all details not only the Ethiopic computus but also the methods of the Christian Easter computus of the 4th century, we can say that these methods contain absolutely nothing of contemporary Alexandrian astronomy which at that time had just reached its final development, of fundamental importance for the next thousand years of mathematical astronomy. The architects of the Alexandrian Easter tables did not use a single concept of pagan astronomy and borrowed all their rules from the simple Jewish procedure to relate the remnants of the Babylonian 19-year cycle by means of "epact" and "saltus lunae" to the Alexandrian civil calendar.

And we now also see how the Jews in the Diaspora in Alexandria regulated their "lunar" calendar during the first centuries of our era. The fierce antagonism against Judaism which is evident in so many ways in our texts guarantees that the data of the Jewish feasts, in particular Passover, were the actual data of contemporary Jewish customs — otherwise the whole construction of the Christian rules would be pointless. This situation changed only centuries later when the Latin West adopted the Alexandrian rules while rabbinical scholarship (in the early 6th century) developed a lunar calendar of much higher astronomical and legal sophistication; in other words, in principle a return to the mentality of the Babylonian astronomers (though not to their level of insight). Since that time Christian and Jewish calendars no longer have had causal connections and the fear of contamination has subsided.

In the introduction to his "Histoire du peuple d'Israël", Ernest Renan wrote : "Pour un esprit philosophique ... il n'y a vraiment dans le passé de l'humanité que trois histoires de premier intérêt : l'histoire grecque, l'histoire d'Israël, l'histoire romain". Having not a philosophically inclined mind, I may perhaps differ from this restriction of interests. For the history of the Easter computus, however, Renan's formulation is unusually well fitted. The mutual antagonism and distrust between the three cultural spheres of Judaism, Alexandrian and Roman episcopats shaped the arguments which are responsible for the form in which the Easter computus still exists today (March 26, 1978).